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ABSTRACT

This paper presents four methods for determining the number of factors to retain: (1) determining an eigenvalue greater than unity (K1); (2) the scree test; (3) the minimum average parallel approach; and (4) parallel analysis. Three of the four methods are illustrated through an example. Although the eigenvalue greater than unity and the scree test are the most widely used methods, the parallel analysis and the minimum average partial are the most accurate methods. (Contains 1 figure, 6 tables, and 19 references.) (Author/SLD)

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Determining the Number of Factors to Retain

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Abstract

The present paper presents four methods for determining the number of factors to retain (eigenvalue greater than unity (K1), scree test, minimum average partial, and parallel analysis). Three of the four methods are illustrated by means of an example. Although the eigenvalue greater than unity and scree test are the most widely used methods, the parallel analysis and the minimum average partial are the most accurate methods.

Determining the Number of Factors to Retain

Suppose a researcher has gathered four predictors and wishes to predict some outcome (dependent variable). There are six simple correlations among the variables. These six correlations are shown as the off-diagonal elements in the correlation matrix in Table 1.

Insert Table 1 About Here

Conceivably, the researcher might be able to visually inspect the correlation matrix and find a pattern or arrive at some conclusion. For example, after visually inspecting the correlation matrix in Table 1, the researcher may conclude that although the correlations only range from 0.377 to 0.535, these correlations may be noteworthy depending on the particular theory being tested. For example, while a low correlation might be important in the medical field, the same low correlation might not be important in the education field. However, as the number of predictors increases, the number of simple correlations increases much faster. In fact, for n predictors, there are n(n-1)/2 simple correlations. Thus, if the researcher had gathered 12 predictors, there would be 66 (i.e., 12(12-1)/2 = 12(11)/2 = 132/2 =66) simple correlations! These 66 simple correlations are shown as the off-diagonal elements in the correlation matrix in Table 2.

Insert Table 2 About Here

Clearly, no researcher can visually inspect such a correlation matrix and come up with a pattern or a conclusion. Consequently, "Some means is needed for determining if there are a small number of underlying constructs which might account for the main sources of

variation in such a complex set of correlations" (Stevens, 1996, p. 362). Another reason for wanting to reduce the number of variables might be that perhaps every predictor is not measuring a different construct. In other words,

> Suppose that we have administered 100 different test of ability and school attainment. In fact, the resulting correlation matrix would consist of positive and often high correlations in the region of 0.5 to 0.6. A factor analysis would reveal that these could be accounted for by a small number of factors: intelligence, verbal ability, and spatial ability. Thus instead of having to look at the scores on 100 tests to understand these correlations, which no human being is able to do, we could understand them in terms of three scores- on intelligence, verbal ability, and spatial ability.
>
> (Kline, 1994, p. 5)

Two methods (a) principal components analysis and (b) factor analysis are commonly used in dealing with this problem. Of the two, "components analysis is the most widely used" (Zwick & Velicer, 1982, p. 253). As Stevens (1996) has noted, "In factor analysis a mathematical model is set up, and the factors can only be estimated, whereas in component analysis we are simply transforming the original variables into the new set of linear combinations (the principal components)" (p. 362).

The purpose of this paper is to present four methods for determining the number of factors to retain. These four methods are (a) the eigenvalue greater than unity (K1), (b) the scree test, (c) the minimum average partial (MAP), and (d) the parallel analysis (PA). To illustrate how to apply three of the four methods, a data set originally analyzed by Holzinger and Swineford (1939) will be used. This data set was collected by

administering 24 psychological tests to junior high school students. According to Hetzel (1996), "This data set has frequently been used by researchers explaining various analytic techniques and is representative of the ability that have been used throughout the history of factor analysis" (p. 177).

Four Methods of Extraction

Eigenvalue Greater than Unity (K1) Method

Zwick and Velicer (1982) noted that "The most commonly employed rule for determining the number of components is to retain those components with eigenvalues greater than 1.0" (p. 254). This rule, also known as the K1 rule, was developed by Kaiser (1960) but can be traced to Guttman (1954). This method is very simple, objective, and easy to use. Moreover, as pointed out by Thompson and Daniel (1996), "This extraction rule is the default option in most statistics packages and therefore may be the most widely used decision rule, also by default" (p. 200).

Eigenvalues are an index of variance explained and can range from one to the number of original variables (when variables are being factored) (Hetzel, 1996, p. 187). Table 3 presents the eigenvalues for the factors for the correlation matrix. According to this table, only the first three components have eigenvalues greater than unity (i.e., the eigenvalue for component one is 2.945, the eigenvalue for component two is 1.760, and the eigenvalue for component three is 1.396). Therefore, using the K1 method, the researcher would only retain the first three components for further analysis.

Insert Table 3 About Here

The percentage of variance accounted for is found by dividing the eigenvalue by the number of variables in the analysis. For this example, the number of variables is nine. Thus, the variance accounted for by (a) the first component is 32.727 (i.e., 2.945/9 = 32.727), (b) the second component is 19.559 (i.e., 1.760/9 = 19.559), (c) the third component is 15.515 (i.e., 1.396/9 = 15.515), and so on. However, since only those components with eigenvalues greater than unity are to be retained, only the first three components are extracted. Therefore, the total variance accounted for using the K1 method is 67.8 % (i.e., 32.727 + 19.559 + 15.515).

Communality coefficients are also an index of variance accounted for but are expressed as percentages. More specifically, "the communality of a variable is that proportion of its variance that can be accounted for by the common factors" (Gorsuch, 1983, p. 29). For example, if the communality were 0.809, the variance of the variable as reproduced from only the common factors would only be 80.9% of its observed variance. The value 0.809 was obtained by summing the square of each of the three components for variable one (i.e., $0.776^2 + (-0.449)^2 + 0.068^2 = 0.809$). The rest of the communalities are calculated in a similar fashion. Table 4 presents the values of the components used to calculate the communalities. The communalities are shown in Table 5.

Insert Tables 4 and 5 About Here

Studies by a number of researchers (Horn, 1965; Zwick & Velicer, 1982; Zwick & Velicer 1986) have evaluated the accuracy of the eigenvalue greater than unity criterion. These researchers found that the number of components retained by the K1 method is often an overestimate. However, Stevens (1996) noted that

> Generally the criterion was accurate to fairly accurate, with gross overestimation occurring only with a large number of variables (40) and a low communalities (around .40). The criterion is more accurate when the number of variables is small (10 to 15) or moderate (20 to 30) and the communalities are high (>.70). (p. 366)

Other researchers have stated that the K1 method may sometimes lead to the retention of fewer factors than should have been retained. For example, Humphreys (1964) concluded that when more components than would have been extracted by the K1 method were subsequently rotated, the results were more meaningful.

In summary, the K1 rule, although commonly used, is believed by some researchers to sometimes underestimate and by many others to grossly overestimate the number of components. Moreover, as pointed out by Zwick and Velicer (1986),

> The use of the K1 rule as the default value in some of the standard computer packages (BMDP, SPSS, SAS) is an implicit endorsement of the procedure, particularly to naive users. This pattern of explicit endorsement by textbook authors and implicit endorsement by computer packages, contrasted with empirical findings that the procedure is very likely to provide a grossly wrong answer, seems to guarantee that a large number of incorrect findings will continue to be reported (p. 439).

Scree Test Method

A second approach for determining the number of factors to retain is the scree test proposed by Cattell (1966). The scree test, available on SPSS and SAS, is based on a graph of the eigenvalues. First, plotting the eigenvalues along the ordinate (y-axis) and the component numbers along the abscissa (x-axis) construct a graph. The graph begins with a steep curve and then plateaus into an almost straight line. According to Cattell (1966),

> This straight end portion we began calling the scree-from the straight line rubble and boulders which forms at the pitch of sliding stability at the foot of a mountain. The initial implication was that this scree represents a "rubbish" of small error factors. (p. 249)

The scree plot for the data being analyzed is shown in Figure 1. Notice that the inclination of the first three components is very steep whereas an almost flat line joins the rest of the components. Thus, there is a break point in the plot. This break point is used to determine which factors to retain. Specifically, retain all those factors that are located before the break point. According to Figure 1, the break point is at component four. Thus, the first three components are to be retained for further analysis. Therefore, using this data set, the K1 method as well as the scree test suggested to retain only the first three components.

A basic idea behind this selection process is that the variables measure a few factors well and a large number of factors much less well. Thus, the predominant factors are large and account for most of the variance whereas the other factors are small, numerous, and account for less variance.

Complications when using the scree test include (a) a gradual slope from lower to higher eigenvalues with no obvious break point in the plot, (b) more than one break point in the plot, and (c) more than one apparently suitable line may be drawn through the low values. Nonetheless, a number of researchers have found the test to be accurate in a majority of cases investigated. Another complication when using the scree test is the interrater reliability. On this, Cattell and Vogelmann (1977) have shown high interrater reliability. But Crawford and Koopman (1979) have reported extremely low interrater reliabilities.

Studies by Tucker, Koopman, and Linn (1969) found that the scree test gave the correct number of factors in 12 of 18 cases. Similarly, Linn (1968) found the scree test to give the correct number of factors in 7 of 10 cases.

Zwick and Velicer (1982) found the scree test to be the most accurate of four methods evaluated across many examples of matrices of known, noncomplex structure. However, four years later the same Zwick and Velicer (1986) concluded that "given these drawbacks and the availability of other clearly superior methods, we can no longer recommend the scree test as the method of choice for determining the number of components in PCA" (p. 440).

Minimum Average Partial (MAP) Method

Another decision rule for factor retention is the Minimum Average Partial (MAP) rule, proposed by Velicer (1976), which is based on the matrix of partial correlations. According to Velicer (1976), the method is exact, can be applied with any covariance matrix, and is logically related to the concept of factors representing more than one

variable. Moreover, the MAP method is expected to produce fewer components than the K1 rule.

Zwick and Velicer (1982) concluded that "A relatively recently introduced method, MAP has not been examined systematically to date" (p. 257). However, Zwick and Velicer (1986) reported that the MAP rule was more accurate and less variable than the K1, Bartlett, or scree methods. Moreover, Zwick and Velicer (1986) concluded "Researchers wishing to ignore relatively small major components should use MAP as a primary method of determining the number of components to retain" (p. 440).

Parallel Analysis

Parallel analysis, conceptualized by Horn (1960), involves the factoring of a second matrix, identical with respect to the number of variables (n) and number of observations (N) as the original matrix, but formed from distributions of random-normal deviates. For example, if one had 1-to-5 Likert scale data for 301 subjects and 9 variables, a 301-by-9 raw data matrix consisting of 1s, 2s, 3s, 4s, and 5s would be generated. Then, the random data matrix is factor analyzed and the corresponding eigenvalues computed. These newly computed eigenvalues are compared to with those obtained from the original data. For any real eigenvalue that exceeds the associated eigenvalue from the random data, its factor is extracted. For example, if the second eigenvalue from the real data is 1.760 and the associated eigenvalue from the random data is 1.274, the second factor would be extracted. On the other hand, if the fourth eigenvalue from the real data is 0.717, and the associated eigenvalue from the random data is 1.021, then the fourth factor would not be extracted. Table 3 presents the eigenvalues from the real data. Table 6 presents the eigenvalues from the random data.

Insert Table 6 About Here

Humphreys and Montanelli (1975) stated that "the parallel analysis criterion is superior to maximum-likelihood as a method for deciding on the number of common factors" (p. 201). Moreover, Humphreys and Ilgen (1969) stated that "the technique is worthy of consideration for use in making the number of factors decision" (p. 578). Zwick and Velicer (1986) concluded "the PA method was the most frequently accurate method examined" (p. 440). Although "the general application of the PA method is problematic at this time because programs needed for its application are not widely available" (Zwick & Velicer, 1986, p. 441), Thompson and Daniel (1996) provide an SPSS program that implements the analysis.

Conclusion

This paper presented four methods for determining the number of factors to retain (eigenvalue greater than unity (K1), scree test, minimum average partial, and parallel analysis). Three of the four methods wee illustrated by means of an example using the data set from Holzinger and Swineford (1939). All four methods reduce a large number of predictors to a small number of factors. Of the four methods, the eigenvalue greater than unity (K1) and the scree test methods are the most widely used, probably because these methods are the default in most statistics packages. However, the minimum average partial and the parallel analysis methods are the most accurate methods.

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Table 1

Correlation Matrix when n=4

Correlation	X1	X2	X3	X4
X1	1.000	0.390	0.395	0.457
X2	0.390	1.000	0.377	0.470
X3	0.395	0.377	1.000	0.535
X4	0.457	0.470	0.535	1.000

Table 2

Correlation Matrix when n=12

r	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
X1	1.000	0.297	0.365	0.441	0.295	0.373	0.293	0.331	0.357	0.067	0.286	0.224
X2	0.297	1.000	0.238	0.340	0.150	0.153	0.139	0.184	0.193	-0.076	0.108	0.092
X3	0.365	0.238	1.000	0.305	0.218	0.212	0.173	0.212	0.239	0.040	0.126	0.177
X4	0.441	0.340	0.305	1.000	0.100	0.159	0.077	0.171	0.198	0.072	0.199	0.186
X5	0.295	0.150	0.218	0.100	1.000	0.657	0.716	0.637	0.739	0.175	0.316	0.165
X6	0.373	0.153	0.212	0.159	0.657	1.000	0.733	0.582	0.704	0.174	0.342	0.107
X7	0.293	0.139	0.173	0.077	0.716	0.733	1.000	0.674	0.720	0.102	0.300	0.139
X8	0.331	0.184	0.212	0.171	0.637	0.582	0.674	1.000	0.582	0.132	0.313	0.184
X9	0.357	0.193	0.239	0.198	0.739	0.704	0.720	0.582	1.000	0.121	0.290	0.150
X10	0.067	-0.076	0.040	0.072	0.175	0.174	0.102	0.132	0.121	1.000	0.447	0.487
X11	0.286	0.108	0.126	0.199	0.316	0.342	0.300	0.313	0.290	0.447	1.000	0.398
X12	0.224	0.092	0.177	0.186	0.165	0.107	0.139	0.184	0.150	0.487	0.398	1.000

Table 3

Total Variance Explained for Real Data

Component	Total	% of Variance	Cumulative %	Extraction Sums of Squared Loadings Total	% of variance	Cumulative %
1	2.945	32.727	32.727	2.945	32.727	32.727
2	1.760	19.559	52.285	1.760	19.559	52.285
3	1.396	15.515	67.800	1.396	15.515	67.800
4	0.717	7.962	75.762			
5	0.629	6.991	82.753			
6	0.535	5.946	88.699			
7	0.478	5.311	94.010			
8	0.289	3.206	97.215			
9	0.251	2.785	100.000			

Extraction Method: Principal Component Analysis

Table 4

Component Matrix

	Component		
	1	2	3
Paragraph Comprehension Test	.776	-.449	.068
Sentence Completion test	.749	-.518	-.029
Word Meaning Test	.762	-.461	.025
Speeded Addition Test	.481	.508	-.361
Speeded Counting of Dots in Shape	.472	.471	-.506
Speeded Discrim Straight and Curved Caps	.534	.315	-.378
Memory of Target Words	.423	-.258	.639
Memory of Target Numbers	.265	.427	.609
Memory of Object-Number Association Targets	.461	.500	.285

Extraction Method: Principal Component Analysis.

a. 3 components extracted.

Table 5

Communalities

	Initial	Extraction
Paragraph Comprehension Test	1.000	0.809
Sentence Completion Test	1.000	0.830
Word Meaning Test	1.000	0.794
Speeded Addition Test	1.000	0.619
Speeded Counting of Dots in Shape	1.000	0.700
Speeded Discrim Straight and Curved Caps	1.000	0.527
Memory of Target Words	1.000	0.655
Memory of Target Numbers	1.000	0.623
Memory of Object-Number Association Targets	1.000	0.544

Extraction Method: principal Component Analysis.

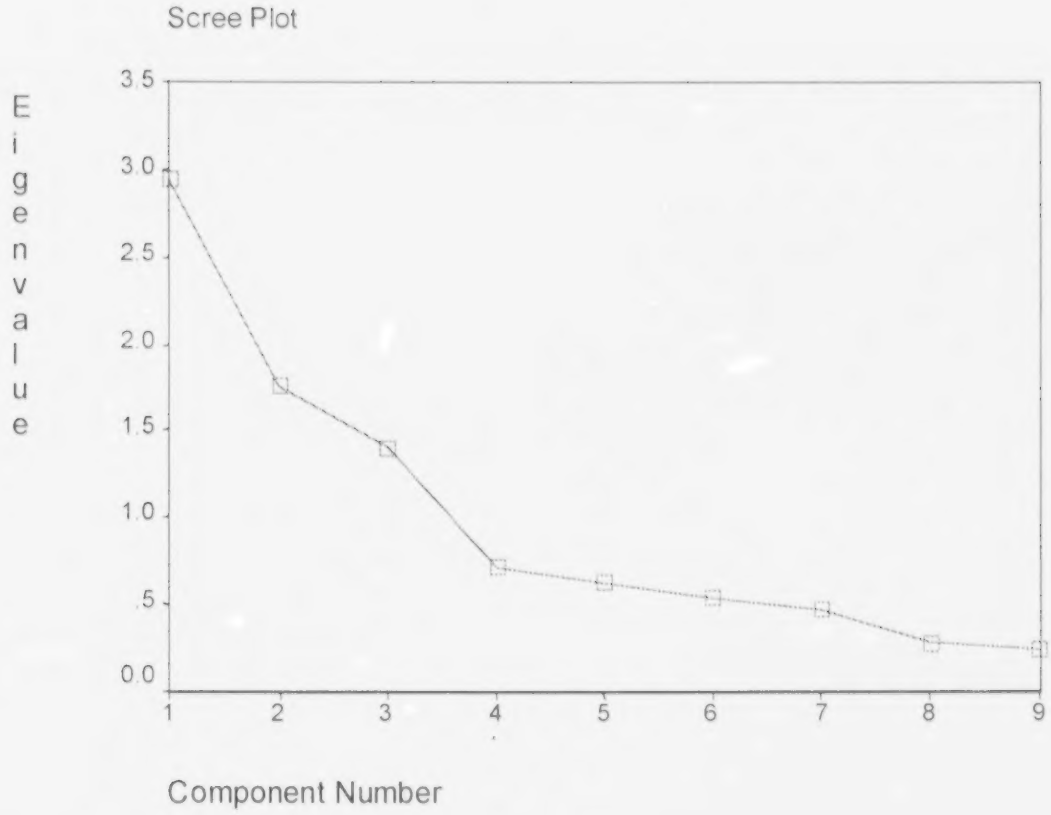
Table 6

Total Variance Explained for Random Data

Component	Total	% of Variance	Cumulative %
1	1.317	14.637	14.637
2	1.274	14.159	28.796
3	1.038	11.530	40.325
4	1.021	11.344	51.669
5	0.975	10.834	62.503
6	0.945	10.503	73.005
7	0.876	9.738	82.743
8	0.812	9.020	91.764
9	0.741	8.236	100.00

Extraction Method: Principal Component Analysis

Figure 1


Scree Plot
Eigenvalue
3.5
3.0
2.5
2.0
1.5
1.0
.5
0.0
1
2
3
4
5
6
7
8
9
Component Number